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FILTRATION OF A COMPRESSIBLE GAS IN AN APPARATUS WITH A DIAPHRAGM

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UDC 532.546.06

Results of an experimental investigation of the filtration of a compressible gas in an axisymmetric apparatus with a diaphragm mounted in the layer of charge or at the exit from the apparatus are presented. A dependence is given to compute the resistance of the apparatus—charge system.

Technological processes realized in an apparatus with fluid or gas filtration through a charge of finely dispersed or bulk materials are extensively used in different industrial branches. The correct organization of these processes depends greatly on knowledge of the aerodynamic fluid motion conditions in such an apparatus.

The authors of [1, 3, 4, 7, 11] investigated the condition of the influence of the side walls on the hydraulic drag of the charge in a uniform flow field of the filtering fluid when examining the charge and the apparatus in combination. In addition to the conditions described above, different nonuniformities at the entrance to and exit from the charge are produced most often in real technological apparatus. A number of papers [2, 5, 6, 9, 10] are devoted to a study of such technological apparatus.

Mathematical models of a low-speed filtration flow in an apparatus are considered in [2, 9]. The resistance of an apparatus with precompression, i.e., when the gas is supplied to or removed from the charge through a hole smaller than the cross section of the apparatus, is experimentally studied on rectangular models in [5, 6, 10].

A number of physical statements of the picture of filtration with precompression directly in a cylindrical apparatus are refined in the present study, and a dependence between the hydraulic drag coefficient and the parameters of the gas stream, the charge, and the apparatus is set up. In comparison to [5], the variation range of the precompression parameter and of the working pressure in the apparatus is hence extended to quantities used in an industrial apparatus of the feeder type in pneumatic transport systems.

The gas flow picture in an apparatus with a charge can be estimated by means of the pressure distribution. The most characteristic pattern permitting an assessment of the filtration flow in an axisymmetric apparatus is the pressure diagram along the apparatus axis. Such diagrams were recorded on a unit (Fig. 1) consisting of interchangeable steel sections with the following diameters: D = 300, 100, 46 mm. Diaphragms with a center hole were used to assure different precompression values. They were mounted both within the charge layer and at the exit from the apparatus. The charge was formed from spherical polystyrene particles. The mean suspended particle diameter d was determined for each fraction obtained between two sieves. The porosity was determined by means of the true and the bulk density of the material [1]. The geometric characteristics of the fractions used in the research are presented in Table 1. The charge was held in the apparatus by a metal mesh with a porosity greater than the layer being studied. Air was used as the filtering agent. The gas pressure at the apparatus axis was measured by means of thin tubes passing through the side wall. U-shaped and pointer manometers were used as recording devices.

Dnepropetrovsk State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 35, No. 6, pp. 980-986, December, 1978. Original article submitted March 9, 1978.

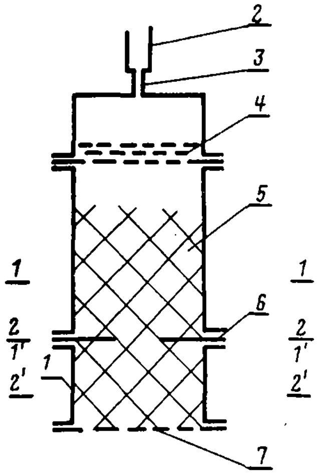

Fig. 1

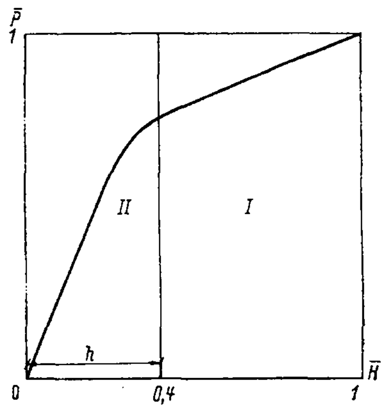

Fig. 2

Fig. 1. Diagram of the experimental set-up: 1) interchangeable sections of the apparatus; 2) air supply prechamber; 3) flow-metering nozzle; 4) levelling mesh; 5) charge; 6) diaphragm; 7) mesh to retain the charge in the apparatus. 1—1, 2—2 and 1'—1', 2'—2' are sections for measurement of the gas parameters for precompression and expansion, respectively.

Fig. 2. Shape of the pressure diagram on the apparatus axis with precompression: I) filtration zone $\bar{f} = 1$; II) zone of influence of precompression $\bar{P} = P^2/P_1^2$.

TABLE 1. Geometric Characteristics of the Charge

Size of sieve cells, m	d, mm	ε
4—2,5	3,06	0,391
2,5—1,9	2,29	0,386
1,9—1,2	1,59	0,38
1,2—0,8	1,12	0,384
0,8—0,6	0,74	0,376

Since the pressure in the apparatus could vary with height from 0-12 gauge atm as a function of the apparatus, charge, and gas discharge parameters, the gas compressibility was taken into account. In this case the gas pressure diagram during filtration was a straight line on the graph $P^2_{atm} = \varphi_1(H)$ [8]. The pressure distribution obtained on the axis of a cylindrical apparatus with precompression was constructed in the same coordinates (Fig. 2). Two filtration zones are shown in the figure. The first corresponds to the filtration modes in the same apparatus, but without precompression, while the second is characterized by higher pressure gradients. Pressure measurements along the radius of the apparatus exhibited a uniformity of the pressure distribution in transverse sections of zone I and spoilage of the uniformity in zone II. This affords a foundation for considering zone I as the zone of influence of precompression. The extent of this latter is denoted by h. The height of the influence of precompression, determined in this manner, was measured in the range $Re_e = 1.5\text{-}10^3$; $\bar{f} = 1\text{-}200$; $\delta = 30\text{-}400$ on apparatus of three diameters. The results obtained show that the height of the influence of precompression is proportional just to the apparatus diameter

$$\bar{h} = h/D = 0.37. \tag{1}$$

The influence of precompression was propagated identically both along and against the stream upon mounting the diaphragm within the layer.

An analogous height of the influence of precompression was determined in [7] for a plane apparatus by means of a pressure diagram recorded from the wall of an apparatus with a charge, and the dependence $\bar{h} = 0.17/\xi_f$ was obtained for $Re_e > 10$. As is seen, for large Re numbers the value of h agrees with the

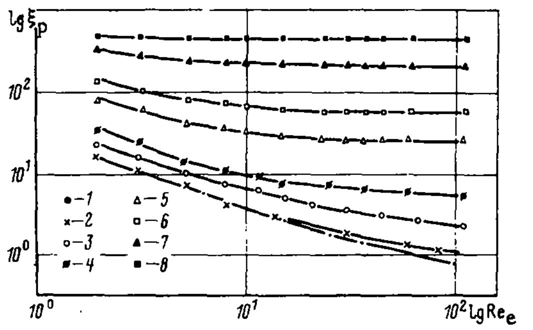

Fig. 3

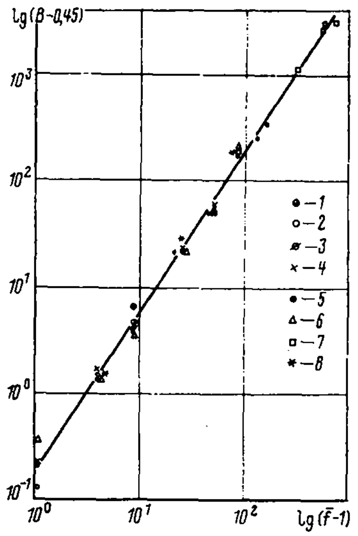

Fig. 4

Fig. 3. Drag coefficient of the zone of precompression influence: 1) $\bar{f} = 1$; 2) 2; 3) 5; 4) 9; 5) 26; 6) 52; 7) 104; 8) 185.

Fig. 4. The dependence $B = B(\bar{f})$ in (7): 1) $\delta = 34$; 2) 46; 3) 65; 4) 94; 5) 140; 6) 400; 7) 140; 8) 140; 1-5) precompression; 6, 7) expansion; 8) diaphragm at the exit from the apparatus.

quantity found for the axisymmetric apparatus. A singularity of the flow picture in plane apparatus is evidently manifest in the dependence of $\bar{h}$ on Re, which was not detected for the axisymmetric apparatus although the range of variation toward small Re numbers had indeed been extended.

The investigations described above permit a separate study of the precompression zone for axisymmetric apparatus since its size is determined uniquely by the size of the apparatus.

The resistance of elements of an apparatus with a charge but without precompression has been studied in a number of experimental papers [1, 3, 4, 7, 11]. All these papers were produced under conditions when the gas compressibility was not manifest or could be neglected. Let us show that the drag coefficient of an apparatus—charge system can be expressed in terms of the coefficient for an incompressible fluid in the case of stream compressibility

$$\xi = \frac{\Delta p}{H} \Big/ \rho \frac{u_e^2}{2g} \,. \tag{2}$$

To do this we write the equation of filtration gas motion in the form

$$\begin{gathered} \partial P/\partial x = k_1 u + k_2 \rho u \sqrt{u^2 + v^2}, \\ \partial P/\partial y = k_1 v + k_2 \rho v \sqrt{u^2 + v^2}, \\ \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0. \end{gathered} \tag{3}$$

Introducing the stream function $\rho u = \partial\psi/\partial y$, $\rho v = -\partial\psi/\partial x$, we obtain

$$\rho \frac{\partial P}{\partial x} = k_1 \frac{\partial \psi}{\partial y} + k_2 \frac{\partial \psi}{\partial y} \sqrt{\left(\frac{\partial \psi}{\partial y}\right)^2 + \left(\frac{\partial \psi}{\partial x}\right)^2},$$

$$\rho \frac{\partial P}{\partial y} = -k_1 \frac{\partial \psi}{\partial x} - k_2 \frac{\partial \psi}{\partial x} \sqrt{\left(\frac{\partial \psi}{\partial y}\right)^2 + \left(\frac{\partial \psi}{\partial x}\right)^2}.$$

If a barotropic process is considered, i.e., $\rho = \rho(P)$, then the pressure can be eliminated from the last two equations and an equation for just the stream function can be obtained. Therefore, in the case of identical boundary conditions, the stream function for a compressible fluid in a barotropic process agrees with the stream function for an incompressible fluid, and hence

$$\rho_i \frac{\partial P_i}{\partial x} = \rho \frac{\partial P}{\partial x}.$$

Integrating this last expression and taking into account that $\rho = P/RT_0$ for $T_0 = \text{const}$, we obtain

$$\rho_i (P_{i_1} - P_2) = \frac{P_1^2 - P_2^2}{2RT_0}. \tag{4}$$

Rewriting (2) in the form

$$\rho \frac{\Delta p}{H} = \xi \frac{G^2 L}{2g}, \quad L = \frac{6(1-\varepsilon)}{gdF^2\varepsilon^3},$$

it can be noted from (3) that

$$\xi_i = \frac{P_1 + P_2}{2P_1} \xi. \tag{5}$$

This equation can easily be obtained for a rectangular channel by the direct integration of the first equation in (3) since v = 0, u = const.

Therefore, the final equation to calculate the drag coefficient with the compressibility and the condition for determining ρ by means of the gas parameters in Sec. 1 taken into account will have the form

$$\xi_i = \frac{\Delta p (P_1 + P_2) gdF^2\varepsilon^3}{H6G^2RT_0(1-\varepsilon)}. \tag{6}$$

As is seen, to determine ξ in an experiment it is sufficient to measure the gas parameters in sections 1, 2 for known parameters of the apparatus, the charge, and the filtering air discharge.

Experiments conducted to determine the resistance of apparatus without precompression for pressure drops to 10 gauge atm showed the validity of (2).

The drag coefficient of just the precompression zone is determined in this paper since the hydraulic drag coefficient of a layer without precompression is known. The diaphragm was mounted within the layer of charge or at the exit from the apparatus. The zone of influence was considered along as well as against the stream with the diaphragm located in the layer and at the exit of the apparatus. In this case the possible appearance of gasdynamic effect in the total drag of the zone was verified since high velocities were achieved in the diaphragm holes in the experiments.

The gas parameters were measured in the diaphragm holes and in the layer at the height h in order to determine the drag coefficient of the precompression zone.

Starting from an analysis of dimensionality, the dependence of the hydraulic coefficient was sought on the following dimensionless parameters

$$\xi_p = \varphi_2(\text{Re}, \bar{f}, \delta).$$

The available set of apparatus, diaphragms, and charge fractions permitted conducting experiments in the following range of parameters: $\text{Re}_e = 1\text{-}10^3$; $\delta = 30\text{–}400$; $\bar{f} = 1\text{-}10^3$.

The experimental data obtained were processed by means of (6) and superposed on graphs in the coordinates $\log \xi_p = \varphi_3 (\log \text{Re})$. One such graph is shown in Fig. 3. The lower curve corresponds to gas filtration in a charge without precompression. It is seen from the figure that the dependence of the coefficient ξ_p for $\bar{f} > 1$ can be sought for $\bar{f} = 1$ in the form $\xi = A/\text{Re} + B$. After the necessary manipulations, approximation parameters were found. The value of the quantity A obtained for all the experiment conditions described is close to the value 36 determined for $\bar{f} = 1$. It is established in an analysis of values of the parameter B that it is a function of just the precompression $\bar{f}$ and is independent of δ in the range examined. The relation between B and $\bar{f}$ is expressed as a power law tending to the known value B = 0.45

for $\bar{f} = 1$. The parameters of this dependence are seen from the graph (Fig. 4). In final form, the approximate dependence of the hydraulic drag coefficient of the zone of precompression influence is written in the form

$$\xi_p = \xi_f + 0.2\,(\bar{f} - 1)^{1.5}. \tag{7}$$

As is seen from Fig. 4, no regular stratification of the quantities B is observed for the versions of the diaphragm location and its induced precompression zone which were considered. The maximum spread in the separate measurements of ξ did not exceed ±15%.

Experimental results in which the magnitude of the ratio was $b/d < 10$ were not considered in seeking dependence (7) since the value of ξ depends substantially on the local porosity in the area of the diaphragm hole and is not reproducible under such conditions.

A dependence of the hydraulic drag coefficient on the governing parameters was obtained in [5] for a plane model of an apparatus with the height H = 0.4 m for the charge and a foundation is given for extending these results to any height of the charge $H > h$:

$$\xi = \xi_f\,[1 + T\,(\bar{f} - 1)\,h/H], \tag{8}$$

where $T = 1 + 48/\delta$.

A detailed examination of the primary results in [5] shows that the constriction parameter $T = \varphi_4(\delta)$ could be neglected in the range of variation $\delta = 100$–620. The value for different T falls on a line $B = \varphi_5(\bar{f})$, excluding the value $\bar{f} = 68$ for which $b/d < 10$. Starting from the above, different values of $B = \varphi_6(T)$ can be considered for $\bar{f} = 68$ as the spread in the experiment under specific conditions. Additional experiments to study the influence of δ on ξ showed that it cannot be taken into account for the values $\delta \geq 20$.

Setting H = h in (8), we obtain a dependence for the precompression zone and we compare it to (7). The difference in the coefficients for the parameter ($\bar{f} = 1$) does not result in any essential discrepancy in the quantities ξ_p calculated by means of (7) and (8) in the range $\bar{f} = 1$–34 examined in [5].

The drag of the apparatus with a charge height H = (4.5–8.5)D and $\bar{f} = 1$–6 was studied on rectangular models in [10]. The deduction made by the authors about the lack of influence of precompression on the total drag of the apparatus does not contradict the results in this paper since the fraction of the drag of the precompression zone was small for the parameters considered.

The investigations performed permit the conclusion that the drag of an apparatus with a diaphragm for the filtration of a compressible gas is the sum of two elements: 1) a charge layer with height H — 0.37D described by (5), and 2) diaphragms with adjoining zones of influence, whose hydraulic drag coefficient is described by (7).

The results obtained can be used to compute the drag of an apparatus with a dense dispersed layer for $Re \leq 10^3$, $\bar{f} \leq 10^3$, $\delta \geq 30$.

NOTATION

D, apparatus diameter; b, diameter of the diaphragm hole; H, height of the charging layer; d, charge particle diameter; ε, porosity; Δp, pressure drop; P, absolute pressure; h, height of the influence of precompression; G, discharge; ρ, density; R, gas constant; T_0, temperature; u and v, projections of the filtration rate on the coordinate axes; x and y, coordinate axes; ψ, stream function; g, acceleration of gravity; F, cross-sectional area of the apparatus; k_1, k_2, A, B, coefficients; Re_e, equivalent Reynolds criterion; ξ, drag coefficient. Subscripts: 1 and 2, gas parameters in appropriate sections of the apparatus; i, filtration parameters for an incompressible fluid; f, filtration without precompression; p, filtration in the precompression zone; $\bar{f} = D^2/b^2$; $\delta = D/d$; $\bar{h} = h/D$.

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APPLICATION OF THE THIRD APPROXIMATION OF THE CHAPMAN—ENSKOG THEORY TO CALCULATING THE THERMAL CONDUCTIVITY OF BINARY MIXTURES OF MONATOMIC GASES AND ESTIMATING THE INFLUENCE OF THE DIFFUSIONAL THERMOEFFECT

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UDC 536.22.083

The coefficient of thermal conductivity and the contribution of the diffusional thermoeffect to the thermal conductivity of binary mixtures of monatomic gases are calculated within the framework of the second and third approximations of the strict kinetic theory. The theoretical results are compared with experimental results.

Inert gases and their mixtures represent the most suitable subjects for testing the strict Chapman—Enskog molecular—kinetic theory, since it treats phenomena occurring in the interactions of molecules without internal degrees of freedom.

Within the framework of this theory the first nonzero approximation for one or another of the transfer coefficients is designated as the first approximation. This is not very convenient, since to obtain the lowest approximations of the transfer coefficients in expansions by Sonine polynomials one must allow for different numbers of series terms: one term is enough for the coefficients of viscosity and diffusion but two terms of the expansion must be taken for the coefficients of thermal conductivity and thermodiffusion. Below, by series approximation we understand the number of series terms in the expansion by Sonine polynomials, it being assumed that the gradients of all the physical quantities are small, i.e., the heat flux and temperature gradient are connected by a linear relation. Thus, the lowest approximation for the coefficient of thermal conductivity is the second: $[\lambda_\infty]_2$.

An adequate amount of experimental data on the transfer properties of nonpolar gases [2-6] and of their binary mixtures [7-13] has appeared in the last decade. Some of the data have an accuracy within 0.1% limits for the viscosity [12] and the thermal conductivity [14], 0.2% limits for the interdiffusion [7, 8], and 1.0% for the thermodiffusion [9]. In a number of reports [15-17] it is shown that a second approximation is inadequate for a more precise analysis of experimental data on the thermophysical properties of gas mixtures. Calculations from higher approximations are especially necessary in the case of mixtures containing very light components [18], as well as when estimating the thermophysical properties of ionized gases [15]. Replacing the first approximation by the second for the coefficient of ordinary diffusion improves its accuracy. But the use of the same approximation for the coefficients of thermal conductivity and thermodiffusion leads to errors of up to 57% for the coefficient of thermodiffusion for an ionized gas [15]. The reason for the considerable disagreement between these data for gas mixtures evidently is the close interrelationship between the processes of heat conduction and thermodiffusion, as well as the use of too low approximations in the theoretical expressions. Nevertheless, there are very few reports devoted to calculations from higher approximations [18,20,21].

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 35, No. 6, pp. 987-993, December, 1978. Original article submitted February 16, 1978.

